Chapter 28
Historia Matheseos: From the History of Phenomena to the History of Ideas. Early Formation of a History of Mathematics

Galina I. Sinkevich

Abstract A mathematical phenomenon or event is the publication of a mathematical document—letters, manuscripts, articles, books. The foundation of the history of mathematics is a scientific biography and bibliography. For two millennia, the history of mathematics presented as a chronologically ordered sequence of mathematical phenomena: "who is after whom." The belief reigned that mathematics is an unshakable body of knowledge, given to us by a Greek miracle. The history of mathematics reached a scientific level in the Age of Enlightenment. A doctrine appeared about historical documents, about the signs of their reliability, ways to distinguish falsification, about ancient written instruments and materials, about styles. For the first time, an understanding of the progress of mathematics arose. The history of mathematics began to be presented by professional mathematicians as the history of concepts, ideas, theories. The article focuses on the evolvement of the history of mathematics as a science and the development of its methodology from the fourth century B.C. to the age of Enlightenment. We consider works of Eudemus of Rhodes, Pappus, Thabit ibn Qurra, Johannes Buteo, Pierre de la Ramée, Joseph Justus Scaliger, Dionisius Petavius, Bernardino Baldi, Joseph Blancanus, Gerardus Johann Voss, Claude-Francois de Chales, Jean Mabillon, John Wallis, Edward Bernard, Joseph Raphson, Christian von Wolff and Johann Heilbronner.

Keywords Historia Matheseos · History of mathematics

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1 Introduction

The history of mathematics was evolving together with the mathematics itself. Its methodology gradually established. Having started with short excursuses in the history of certain issues and biographical details of scientists, the history of mathematics ended up with research using historical, textual, and mathematical methods, and achieved significant results. One would find wealth of diverse literature devoted to the history of mathematics—from popular literature to rigorous research. People in all countries studied the history of mathematics; it is included in education courses; it is interesting for all lovers of mathematics. The fruitful period after 1758 began with the appearance of *Histoire des mathématiques* by J.-É. Montucla, it is very well studied. In the eighteenth and nineteenth centuries, the main scientific approaches to the history of mathematics were formed, they represent a move to a more systematic history of mathematics. It was a stage for later works. The new rigorous approaches to historiography that developed in the nineteenth century in general to some extent affected history of science as well. But before the age of Enlightenment the history of mathematics was in its formative phase, which was less studied. Our goal will be to give an overview of the historiography of the history of mathematics in the early period before 1758.

2 4th Century B.C., Eudemus of Rhodes

Mathematics was laid down as a science in Ancient Greece, and the first famous work devoted to the history of mathematics was the *History of Geometry* (Γεωμετρικὴ ἱστορία) by Eudemus of Rhodes,[1] Aristotle's student. This work was repeated in *The Philosophical and Mathematical Commentaries of Proclus on the First Book of Euclid's Elements* by Proclus (The Catalogue of Geometers; Diadochus 2013, pp. 82–86). We do not know the historiographical style of Eudemus. Undoubtedly, he has undergone changes in the retelling of Proclus. As J. Mansfeld wrote,

> The ideas of earlier philosophers were used and interpreted in many ways, and, more often than not, served merely as springboards […]. Surveys of earlier philosophers and philosophies and even anthologies containing purple passages were also composed for the delectation of a more general public, but the doctrinal contents of such works as well as the selections that were made, though containing mostly traditional material, were often updates and reflects the interest and predilections of their times, which as a rule were indebted to those of the professional philosophers. The transmission of the views of the early Greek philosophers (the so-called *physikoi*) therefore is not only quite fragmentary but also often coloured or even biased. (Mansfeld 1999, pp. 22–23)

Eudemus is known to have also written *The History of Arithmetic* and *The History of Astronomy,* none of which has survived to our days and are only mentioned by

[1] *Geometry and Geometers before Euclid* with a fragment from Eudemus of Rhodes was first published in Russian in a journal of A.I. Goldenberg, "Mathematical Leaflet", v. I, 1879–1880.

other authors. He concluded that arithmetic resulted from trade activities of the Phoenicians, and geometry, from geodetic activities of Egyptians. He described a period of three centuries in Greek mathematics in his works. According to L.Ya. Zhmud', researcher of the ancient world, Eudemus laid emphasis on three aspects: he specified the "ground breaker", looked for a faithful mathematical proof, and compared works of several scientists who studied the same issue (Zhmud 1997), p. 277).

> The main question he raised in his *Histories*, was 'who discovered and what? (Zhmud 1997, p. 289).

Eudemus arranged his materials chronologically—from discoveries of the teacher to discoveries of disciples. Proclus began classifying mathematical literature by goal: "subject of investigation and for students" (Diadochus 2013, Introduction, Chap. 7, p. 87). Therefore, he compared teachers' merits as well.

Whereas the attitude to discovery of mathematical results was heuristic, meaning that ancient mathematicians 'discovered', not 'created', the history of mathematics was presented as a history of discoveries, i.e. as a linear flow 'who follows whom'.

Works of ancient mathematicians was normally preceded by a historical approach, for example, in Archimedes' works (third century B.C.). Starting with Diogenes Laertius (2nd–third century), biographical compendiums began to appear, i.e. summary versions of classic authors with comments which also included elements of history of mathematics. *Mathematical Collection* by Pappus of Alexandria (3rd–fourth century) were a most complete compendium translated into Latin by Federico Commandino in 1588. It is thanks to Pappus that we now know many ancient problems. Let me also mention a compendium of Eutocius of Ascalon (5th–sixth century) which along with fragments from *The History of Geometry* by Eudemus of Rhodes contains solutions of the problem of cube duplication (Delosian problem) by ancient mathematicians; and Simplicius' philosophical works (5th–sixth century). The genre of collections of rendered works (compendiums) became predominant in the Late Antiquity and Middle Ages up until the early Modern Age. Their subject was not only Greek mathematics, but Byzantine (e.g. treatises of a Byzantian, Maximus Planudes (thirteenth century)), and a little of Indian mathematics as well. Since the chronological framework of such works was not broad, and all the characters lived in the same region, the sequence of events was indicated according to the principle "who follows whom", or the year of the Olympiad was called, or the name of the ruler.

Thabit ibn Qurra[2] (836–901), a mathematician and astronomer from Bagdade, translated works of Archimedes, Apollonius, Euclid, Ptolemy, and other ancient writers into Arabic. We can only read his translations of Archimedes' Treatises *Book of the Construction of the Circle Divided into Seven Equal Parts, On Tangent Circles*, as well as Books V–VII of *The Conics* by Apollonius. In Arabic mathematical literature of 8th–thirteenth centuries, Arabic translations of ancient classic authors were accompanied by comments and supplemental information intended

[2] Sometimes, in Russian literature his name is spelled as Qurra.

to systematise the knowledge of Greece, India, and the Arab world. Mathematicians' biographies were arranged chronologically in the Collections with extensive catalogues of manuscripts accompanied by historical comments. For example, in his *Introduction on Superiority of the Science of History* (*Prolegomena* or *Muqaddimah, Introduction to History*), an Arab historian Ibn Khaldun (1332–1406) reviewed the development of Greek and Islamic mathematics, dating its stages by periods of reign of tsars and khalifs.[3] The treatise of Kâtib Çelebi (Katib Çelebi 1835–1858), Turkish historian of the seventeenth century, *The Removal of Doubt from the Names of Books and the Arts,* containing 14,500 book titles and 10,000 names of authors and scholiasts in alphabetic order, which was published in Leipzig in seven volumes in Latin (Kâtib Çelebi 1835–1858), is a most vivid example of bibliographic encyclopaedic genre.

As of the twelfth century, translations from Arab and Greek, or renderings of ancient mathematicians, and comments containing historical data, began to appear in Europe. Euclid was first translated or rendered by Adelard of Bath (twelfth century), an English scholastic philosopher who also translated al-Khwarizmi's astronomical tables and renderings of Euclid's Elements in 15 books by Giovanni Campano (Campanus, Campani, deceased in 1296). Campano himself did not know Arabic. He made his renderings with comments based on earlier translations.

With advent of book printing in late fifteenth century, Greek and Latin texts of Archimedes, Euclid, and other ancient authors started to appear. Scientists began to compare translations, analyse translators' and scholiasts' (relaters') errors.

3 1559, Johannes Buteo

In 1559, a book of J. Buteo[4] (Buteo, Jean Borrel, 1492–between 1564 and 1572), *Quadrature of the Circle,* was published in two books where he invalidated many quadratures and sheltered Archimedes from hostile criticism; he also provided a list of errors made by Campani, Zamberti, Finé, and Peletier in their interpretations of Euclid in Latin (Buteo 1559). Buteo analysed the errors of these translators of Euclid and Archimedes; provided approximate calculations of Bryson of Heraclea, Archimedes, and Ptolemy; and criticised the common misconception originating from Zamberti to the effect that the author of the demonstrations in Euclid's elements was Theon of Alexandria.[5] *Buteo has confidently mastered Archimedes' method and*

[3] This book is available in French (Les Prolégomènes d'Ibn Khaldoun 1863).

[4] This is the Buteo which in 1559 calculated the capacity of Noah's Ark.

[5] Campano was the author of one of the first renderings of Euclid into Latin (*The Elements* in 15 books). Italian Zamberti (Zamberti, Zambertus, 1473 – after 1543) was the first to publish a printed translation of Euclid from Greek in 1505 (The Elements and other books). Zamberti corrected the mistakes in the medieval Campano's version in Latin. However, Zamberti was not a mathematician. Therefore, Luca Pacioli criticised him for his assaults upon Campano. In 1543, Tartaglia published his first translation of Euclid considering the text of Campano and Zamberti. Zamberti believed that Theon was the true author of the proof, while those were only the definitions and statements which

provided a summary of its use in the ancient world and Middle Ages (Beckmann (1971–2015, p. 97).

4 1567, Pierre de la Ramée

Buteo's book, as well as the book of Ramus (Pierre de la Ramée, 1515–1572) contains no dates, adhering to the principle 'who follows whom'. In 1567, Ramée published his *Introduction to Mathematics* divided into three books (Ramée 1567) in Latin which formed a preamble to his large work, *Thirty-One Books of Mathematical Essays* (Ramée 1569). In the sixteenth century, the idea of comparing, researching and verifying the information communicated by predecessors arose, and a new criterion of scientific knowledge was gradually forming, connected with observation and practice. Poor historical material, which Ramée had, serves him only as a springboard for attack to the old methods of teaching. The fact that the exposition of mathematics has not changed in two thousand years is, for Ramée, proof of the "complexity and ambiguity" of the subject, although, of course, it does not diminish the value of the "Elements". But, despite the high authority of Euclid, it is necessary to subject them to research, removing the "non-methodical" shortcomings: introducing definitions of mathematical concepts before they become necessary, teaching first geometry, and then algebra. According to Ramée, it is not knowledge that develops, but only the way of its presentation, teaching.

This was a summary of prior discoveries in mathematics divided into three periods: from Adam to Abraham (Chaldean); from Abraham (Egyptian period); from Thales to Proclus and Theon of Alexandria (Greco-Roman period); and the fourth, modern period, from Theon (fifth century) to Copernicus, Regiomontanus, and Cardano. The first book of the Introduction (pp. 1–39) describes the first three periods and lists 65 names of Greek mathematicians, and the whole of Greek and Roman science is presented on 36 pages. The second book – classification of mathematical sciences (described only arithmetic and geometry as a mathematical science; as to astronomy, optics, and music, he assigned them to physics) and their development in various European countries (pp. 39–71). He listed around 30 names of sixteenth century Reformers in his second book. They were mostly theologians, translators and scholiasts. Of mathematicians and astronomers, he mentioned, inter alia, Regiomontanus (but young Tycho Brahe is absent, Ramée got to know him two years later); sixteenth century: Herlinus, Copernicus, Rheticus (Copernicus' student), Rheinold,

belonged to Euclid. A French mathematician and cartographer Orontius (Oronce Finé, Orontius Finnæus, or Finæus, French Oronce Finé; 1494–1555) was Buteo's teacher. In 1532, he published his Protomathesis (Introduction in Mathematics) in Paris, where he explained the main notions used in The Elements by Euclid and the calculation of areas of planar figures as provided in Archimedes' works. He also described in this book his method of solving the circle quadrature problem which was later criticized by his student Johannes Buteo. Buteo criticized all the above authors including his teacher.

Santbecus, Leovitius (Cyprian Karasek Lvovicky), Dasypodius, Clavius, Landtgravius, Morshemius, Grunius, Xylander. No mention of Stiefel. Perhaps this is due to Ramée's negative attitude towards hypotheses as a scientific tool, as well as to Stiefel's attempts to combine the "scientific with the miraculous" (Cantor 1857, p. 364).

Ramée noted that the copies of ancient manuscripts were preserved in Florence thanks to the Medici family,[6] which evidences of his good knowledge of Italian historical literature. He set forth his speculations on the changes in methods of teaching mathematics in Christian Europe, giving preference to teaching in Protestant universities and criticising Aristotle. He mentioned Latin translations of Euclid and dissemination of information on Greek mathematics in Christian Europe. However, he mentioned nothing of the development of mathematics in the Islamic East. As in early works devoted to history of mathematics, Ramée considered mathematics as a combination of ancient Greek achievements remaining unchanged until the seventeenth century to be looked up to, sometimes criticising teaching methods. He told nothing about Kepler's, Cardano's, or Tartaglia's results, although mentioned their names. There was no evolution dynamics of mathematics or its contents in Ramée's. In the emerging Age of Enlightenment, the notion of the progress of science had not yet become a historical category.

The third book described the development of teaching mathematical methods in European universities. The purpose of mathematics was described as practical application in trade, physics, architecture, astronomy, and other areas (Ramée 1569, pp. 71–107). The book contained almost no historical information—only rhetoric on the way ancient classic mathematicians should be presented in educational institutions. The book mentioned sixteenth century mathematicians, such as Cardano, Maurolico, Piccolomini, Commandino, Tartaglia, and Dürer. Ramus sets out his main views on teaching. According him, first of all, the general formulation of the problem should be given, after which definitions of basic concepts should be introduced; then the problem is divided into its component parts, each of them is defined, and, finally, explanations are made using illustrative examples. Ramus sharply criticized not only Aristotle, but also Euclid, reproaching him in the absence of a method. He called science arts: for example, arithmetic is the art of counting well, geometry is the art of measuring well. New methods should be based on the conscious assimilation of knowledge and promote the development of students' independent thinking. In the textbooks of Ramus himself, only the most necessary theoretical information is given, the main content is given to a large number of examples. He consistently pursues the idea that the main source of scientific knowledge is practice, it and only it verifies the authenticity of the theories. It is necessary to revise all the sciences, to abandon the system of disputes. Mathematicians of 16–17 centuries recognized for the textbooks of Ramus "Arithmetic", "Algebra" and "Geometry" high methodological advantages. This created the glory of Rama "great enlightener", but gave rise to many enemies.

[6] Ramus' book was devoted to his patroness Catherine de Medici, the Queen of France. However, it did not help to retrieve the Huguenot from ruin on the St. Bartholomew's night.

5 New Chronology

Chronology was an important problem in the history of mathematics as well as in general history. Each culture had its own chronology, and history of each culture did not correlate with other cultures; social time was described in each culture regardless of others. In Greece, they dated events by Olympiad; in the Arab world, from Hegira and by khalif; in Rome, time keeping was 'from the founding of Rome' (ab urbe condita); in Byzantium, 'from Adam', i.e. 'since the creation of the world'. Social time in different cultures was autonomous. The errors were significant. In 1582, the Pope, Gregory XIII published a bulla, *Inter gravissimas,* with an invitation to switch to a new calendar. First, some Catholic countries began gradually switching to the Gregorian calendar; thereafter, over a period of 17th—eighteenth centuries, Protestant countries, including Great Britain in 1752, were switching to it. Russia switched to the new calendar in 1918. It was Dionysius Exiguus who in the sixth century suggested keeping time 'from the year of our Lord' (ab Anno Christi, ab inscriptione, Anno Domini). In Europe, this way of time keeping spread in late Middle Ages. In 725, in addition to the time keeping by Olympiad and by emperor, Bede the Venerable introduced absolute chronology for the first time. Beginning Chapter Two of Book One of his *Ecclesiastic History of the English People* (Historia ecclesiastica gentis anglorum), Bede wrote in 731: "ante incarnationis dominicae tempus" (before the incarnation of our Lord). This was the first time ever that the time countdown was mentioned. This is not to say that the countdown scale—B.C. and A.D.—developed in a prompt and natural way. Before the sixteenth century, along with 'A.D.' time keeping, they used 'anno mundi' and many other ways of reckoning.

The historical research space has expanded not only geographically, but also chronologically. There was a need for a unified time scale.

In the period from 1583 to 1629, books of Joseph Justus Scaliger (1540–1609) were published. He was a connoisseur of ancient culture and ancient calendars and the founder of modern chronology as an auxiliary science of history. Scaliger found ways of conversion between the systems of Ancient Rome, Ancient Greece, East Asia, and Mexico, using the method of astronomic dating of events by eclipse. This enabled him to correlate scientific discoveries in various cultures in the course of time.

In 1627, Dionisius Petavius (Denis Pétau, 1583–1652), a French scholar, suggested a 'before Christ' (ante Christum, B.C., century before Christian Era) system of counting down dates.[7] This system was universally recognized by the end of the eighteenth century.

[7] It should be noted that it's in these years that a new understanding of a number scale comes into existence too: minus (negative) numbers, as numbers that are less than zero (according to Stifel), are located to the left of (behind) zero. In 1629, A. Girard wrote about negative solutions of equations: "Solution using minus is explained in geometry as reversion, and minus retreats where plus goes further." (Descartes 1637, p. 228).

6 Bernardino Baldi

Bernardino Baldi (1553–1617), an Italian poet and mathematician, Commandino's student, was the first to try and use the new chronology in combination with former traditions of presenting history as a chronicle. It took him 12 years to create a *Chronicle of Mathematicians including their curriculum vitae* (Baldi 1707) as basis for a more comprehensive work. Among other preparatory materials for this work, his writings about Pythagor, Ctesibius, Hero of Alexandria, and Copernicus were preserved. The *Chronicle of Mathematicians* was written in Italian and contained around 200 life histories and an index of names. The book was written as a popular Who's Who. Names were arranged in a chronological order; in the left field, Greek chronology by Olympiad; in the right field, years before Christ or after Christ. E.g. about Euclid: 122 (122nd Olympiad) in the left field, 290 (anni avanti Christo – 290 before Christ) in the right field; and a text as follows:

> *Euclid*. There is some evidence that the most esteemed mathematician from the City of Gela in Sicily studied in Alexandria and probably in Athens. We has written a lot, that is to say a book entitled *The Elements of Geometry* in which he surpassed all those who wrote before him, and he was so glorious that he was named στοιχειωτής – Stichiota[8] (Sic! – G.S.). In addition to the *Elements,* he wrote a book entitled *Data*, *Porisms* in three volumes about perspective projection, about mirrors, a book about phenomena, and, to the best of my knowledge, a book entitled *Conics*, instead of the book about basics of music erroneously assigned to him. There is another apocryphal book about division of surfaces assigned to him by Mohammed from Bagdad. There is also a Plato section of Euclid which, according to Proclus, prepares the use of *The Elements* for the purposes of plotting Platonic solids. (Baldi 1707, pp. 22–23)

Baldi also mentioned Arab and Persian mathematicians, as well as mathematicians from North Europe; many astronomers; some philosophers and theologians (e.g. Thomas Bradwardine, Nicholas of Cusa, Abraham Zacuto). Notably, there was no article devoted to Girolamo Cardano, although he mentioned his name in the articles devoted to Swineshead and Tartaglia. The timeframe covered by this book was from 545 B.C. to 1596 A.D. Since Baldi included Muslim mathematicians in his chronicle, he needed a unified time scale.

7 1615, Joseph Blancanus

In 1615, in his *Thesis on the Nature of Mathematics,* Joseph Blancanus (Giuseppe Biancani, 1566–1624), Italian mathematician and astronomer, represented the history of European and Asian mathematics as a history of discoveries in accordance with the new chronology, *Chronology of famous mathematicians* (Blancanus 1615), in Latin.

[8] The Author of "The Elements", Euclid the stoicheiotes.

Although it contained certain inaccuracies,[9] it was more complete than Ramée's work and included Islamic mathematicians. This was an attempt to use a unified time scale representing the European and Arab history of mathematical discoveries, so Biancani uses the new timeline.

8 1650, Gerardus Johann Voss

The book which enriched the history of mathematics with new methods was written by a Dutch historian and philologist, Gerardus Johann Voss (Vossius, 1577–1649). His materials on the history of literature were such extensive that they included information on the development of mathematics as well. He collected these materials in his work On the Nature and Structure of All Mathematical Sciences Supplemented by Mathematicians' Chronology published posthumously (Voss 1650) and republished as a part of the book entitled *On Four Main Arts, On Philology and Mathematical Sciences, Supplemented by Mathematicians' Chronology*, issued in three books (Voss 1660). Voss was not a mathematician and, at times, used inaccurate information, for which he was reasonably blamed by researchers. However, he was the first to use philological and source study methods in his historical and mathematical review.

Voss began with the history of alphabetical and digital numbers and symbols, systematised his presentation by section (geometry, arithmetic, optics, music, mechanics, logistics, geodesy, astronomy, calendar, chronology). Having mentioned Greek mathematicians, among others, he mentioned such mathematicians as Boethius, Alcuin, al-Farghani, Ibn al-Haytham, Sacrobosco, Nicholas of Cusa, Regiomontanus, Zakuto, Dürer, Copernicus, Maurolico, Cardano, Gemma, Commandino, Mercator, Ramée, Clavius, Viète, Ludolph van Ceulen, Tycho Brahe, Neper, van Roomen, Grégoire de Saint-Vincent, Stifel, Mersenne, Snellius, J. Golius, Cavalieri. He placed emphasis on translations of Greek classic authors into Arabic and thereafter, from Arabic into Latin (Voss 1660, p. 55); addressed works of Arabian historians. In the section devoted to the history of Alfonsine tables, Voss mentioned the notion of a progress of science for the first time—"progress of astronomy after Greeks" (Voss 1660, p. 146). For him, publishing a book, including a translation, or annotation, was a mathematical event. Voss' book was followed by an index rerum & verborum, i.e. an index of objects and words, an index of names and subjects with page numbers, and in addition, a list of printing mistakes. All this suggested a new sample form of a historical and mathematical research.

[9] E.g. Thābit ibn Qurra (836–901) was described as a thirteenth century scientist; Roger Bacon (thirteenth century), as a fourteenth century scientist; Leonardus Pisanus (Fibonacci, early thirteenth century), as a fifteenth century scientist.

9 1674, Claude-François De Chales

French mathematician, Jesuit professor de Chales (Dechales, 1621–1678)[10] was the first in historiography to express his consciousness of the advance of mathematics in his *Treatise on the Advance of Mathematics and on Famous Mathematicians* which made part of Volume One of his three-volume edition of *The Course and World of Mathematics* (Chales 1674), a pansophy which contained information from mathematics, physics, astronomy, astrology, and architecture. De Chales translated Euclid, and this translation was popular in France although it was worse than Roberval's translation. D. E. Smith wrote that, although de Chales published Euclid, his own contribution in the subject was more than modest (Smith 1951, p. 386).

10 1681, Jean Mabillon

In 1681, a book of J. Mabillon (1632–1707) was published. Historian, founder of the discipline of palaeography, historical criticism, and chronology, wrote a book entitled *Diplomacy in Six Books* (Mabillon 1681). Mabillon understood diplomacy as a science addressing historical documents, evidence of their accuracy, methods of identifying forgery, ancient written instruments and materials, styles. Mabillon's book contained engraving plates with examples of ancient writing. Voss' and Mabillon's works influenced subsequent researchers and J. Wallis, in the first place.

11 1685, John Wallis

His *Treatise of algebra both Historical and Practical. Shewing, the Original, Progress and Advancement thereof, from time to time; and by what Steps it hath attained to the Heighth at which now it is. With some additional Treatises. London. M.DC.LXXXV* (Wallis 1685) was published in 1685. Historians of mathematics (Cajori, Bobynin,[11] Popov) blamed him of nationalism and lack of personal modesty which consisted in attributing discoveries of other mathematicians to himself (or his compatriots). This was a just reproach which, however, did not make his treatise less interesting, as it was written by an outstanding mathematician. The history of mathematics (algebra) here was for the first time presented as a history of ideas. Speaking of works of ancient classic authors, Wallis gave names of translators and publishers.

[10] Not to be confused with geometer Michel Chasles (1793–1880), French mathematician (geometer) and historian of mathematics.

[11] Victor V. Bobynin (1849–1919), Russian scientist, teacher, historian of mathematics, professor at Moscow University; one of the authors of the Encyclopedic Dictionary of the Brockhaus and Efron Publishing Society; publisher of the first Russian journals on the history of mathematics (1884–1905).

But it appears from the following example that the Arab history (chronology) had still existed apart from the European history. Wallis believed that Arabs knew algebra:

> After Diophantus (if not before, also) this Learning was pursued by *Arabic* Authors (but little known in *Europe* for a long time) […] Divers writers (is said) there are of *Algebra* in that Language, and from them (I suppose) the Denominations of *Diophantus* (if from him they learned it) came to be changed; and (beside the Denominations of Root, Square, and Cube) that of Sursolide (first, second, thirds, etc.) introduced. But I rather think the *Arabs*, either of themselves, or from some others, had it long before *Diophantus*, and think this reckoning of *Powers* (by Sursolids, etc.) different from Diophantus. (Wallis 1685, p. 5; Author's Italics)

According to Wallis, in England, algebra started to develop in the twelfth and thirteenth centuries—earlier than in Europe—thanks to the fact that English scholastics knew Arabic. Englishmen used to visit Spain and bring many mathematical manuscripts with them. For example, in 1180, J. Morley (Morlacus, Morley, around 1140—around 1210), mathematician and astronomer, studied Arab mathematical manuscripts in Toledo and brought a valuable set to England. Wallis believes that the English were the first to translate Greek mathematical texts from Arabic. For example, in 1130, Adelard was the first to translate Euclid's Elements. Wallis mentioned an English theologian and historian, monk Bede Venerable (Saint Beda, Beda Venerabilis, late 7th—early thirteenth century), who wrote the history of English people; and then Alcuin (Alcuinus, around 735–804). Wallis erroneously called him Bede's student.[12] Wallis told in detail which Arab translations of ancient authors were brought to Oxford (including Merton College) and translated into Latin and English. In fact, this was a rendering of Voss' history of mathematics in the context of English history. He further told about the numerical values which originated from Moors and Arabs (Wallis 1685, p. 7) and Maximus Planudes. He also considered other number notations—Roman and Greek literal and number notations worldwide. He admitted that, although Arabic figures came from Saracens and Arabs, they originated from India. He compared Sacrobosco's (Johannes Sacrobosco, Sacrobosco, John of Holywood, around 1195–around 1256) numbering, who described fundamentals of Indo-Arabian numbering and arithmetic in his treatise *Algorithm* (*Algorismus de integris*): operations of addition, subtraction, averaging, duplication, multiplication, division, summing up arithmetical progressions, rooting, and cube-root extraction. Wallis believed that it was Luca Pacioli who was the first to bring the new notation to Europe (Luca de Burgo, Luca Pacioli, Summa de Arithmetica, Geometria, Proportioni, et Proportionalita, 1494). Thereafter, Wallis divided his narration into topics, which tells the difference between him and his predecessors.

Further Wallis' synopsis:

Astronomic tables: Ptolemy, Copernicus. Decimal fractions appeared; then, logarithms. Archimedes' methods, including the method of using big numbers (with

[12] Alcuin was born after Bede had died, and was in tutelage of Archbishop Egbert, Bede's student.

the help of 60-ary fractions, e.g. ¼ = 15′). Operations on them. On decimal fractions and use thereof in certain branches of arithmetic; on antiquity of decimal fractions. Described works of Briggs, Oughtred,[13] Gellibrand[14] (*Trigonometria Britannica*, 1633), Regiomontan (1464), Ramus (1560), Schoener[15] (1585), Record (1550), Stevin (1585). Reducing fractions, or proportions, to a smaller number of characters with the nearest approximation to the real value: the development line from Archimedes: Van Gulen, Snellius.[16] Chapter 11: application of the same in relation to the proportion of the diameter and length of circumference (from Archimedes).

In Chap. 12: *On Logarithms*, Wallis wrote about Neper, Briggs, Kepler, Rudolphian tables (1627), Mercator's *Logarithmotechnia* (1668), however, he did not mention logarithmic tables of J. Speidell[17] (Speidell 1619, 1622; Hobson 1614, p. 43) or the logarithmic rule invented by the English (astronomers Gunter and Wingate, mathematician Oughtred).

In Chap. 13, Wallis spoke about algebra:

> From the Arabs or Saracens, together with their Algorism by the Numeral Figures, (and other parts of Mathematical Learnings) we received also our *Algebra*, brought into *Europe*, partly be the way of *Greece* (as may seem by what we have of *Maximus Planudes,*) and partly by the *Moors* in *Spain.* Whither I find, that divers of our *English* Mathematicians (about the Twelfth Century) did resort, on purpose there to learn from the *Moors*, not only the *Arabic* Language, but especially the Astronomical and other Mathematical Learning. And this (no doubt) of Algebra amongst the rest; though I have not yet seen any thing of Algebra in any Ancient Manuscripts. (Wallis 1685, p. 61; Author's italics and orthography)

About Leonardo Pisano, Luca Pacioli, Cardano, Tartaglia, Nunes, Bombelli, and other authors of *Algebra* before Viète. He told a lot of good things about Pacioli (Pacioli's own books and his translation of Euclid).[18] In Part Five of his *Sum*, Pacioli provided the basic materials on arithmetic as provided by ancient authors and his contemporaries. Subsequently, Wallis rendered Voss' story of Leonardo Pisano. Lucas de Burgo was the first to describe the abacus method, i.e. that of Luca Pacioli. In page 62, he mentioned Stifel, his *Arithmetic* of 1544, Rudolph, Cardano's *Arithmetic* and his *Great Art* (*Ars magna*) of 1545. Cardano's rule of solving a cubic equation which, according to Cardano, was found by Tartaglia too. Cardano's student Luigi Ferrari also added a bit.[19] In 1567, Pedro Nunes published *Algebra* in Spanish.[20]

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[13] William (Guilelm) Oughtred, 1575–1660.

[14] Henry Gellibrand, 1597–1636, Professor of Astronomy in Oxford who completed Briggs' unfinished work.

[15] Lasarus Schonerus, Schoener, Schöner, 1543–1607, Ramée's publisher, scholiast, and partly a contributing author. He taught mathematics in Neustadt; was Provost in Schmalkalden, Thuringia; and taught mathematics in Corbach high school.

[16] Willebrord Snellius,1580–1626, Dutch mathematician and astronomer.

[17] John Speidell, 1600–1634, teacher of mathematics in London, drafter of logarithmic tables.

[18] Wallis called this period the Italian period. However, in effect, this was a revised publication of the Latin translation of Euclid made by Campano, where Pacioli had corrected numerous errors.

[19] This is what Wallis wrote. However, this 'a bit' was a solution of a 4th-degree equation.

[20] Wallis was mistaken here. It was in Portuguese.

In 1579, Rafael Bombelli published the treatise *Algebra*[21] in Italian. In this book, he published the rule of solving a cubic and biquadratic equation as Tartaglia and Cardano had done before (Wallis 1685, p. 63).

We believe that Bobynin's observation to the effect that Wallis took the credit for the solution of the irreducible case of a cubic equation (Popov 1920, p. 149) is groundless. On pages 173–174, Wallis spoke of the similarity of Cardano's and Harriot's methods. Further, in Chapters 46–48 (p. 175–181), he presented his own method of cube-root extraction from a binomial in the form.

$$a + \sqrt{b}, \text{ where } b \text{ may have any sign.}$$

He found this method in 1647–1648, and it was really similar to Bombelli's method published in 1572, and Wallis was familiar with Bombelli's book.

Then, Pierre de la Ramée (1570) was mentioned. The book was published by Schoener who also wrote books (*Numerical Geometry*). Leonard Digges[22] was one of our (i.e. Wallis') compatriots who wrote a book in 1579 entitled *Stratioticos* (military message deliverer). Another was Robert Recorde, 1552.[23] Chapter 14 was devoted to François Viète and his *symbolic arithmetic*.[24]

Chapters 15–29 were devoted to Oughtred (Wallis was his disciple) and his book *The Key to Mathematics* (*Clavis Mathematicae*; Oughtred 1631), a textbook of arithmetic which was republished three times in Oughtred's lifetime and was thereafter used even in the eighteenth century. Wallis rendered it in the tiniest detail. Beginning with Chap. 30, Wallis wrote about Harriot's *Algebra* (Harriot 1631), rendered it in detail, and asserted that Descartes adhered to Harriot. In this book, Harriot showed the way algebraic equations were laid down by multiplying linear binomials for the first time. Harriot rule (as Euler set it forth with reference to Harriot[25]) was as follows: each equation has as many positive roots as many variations of sign it contains, or as many negative roots as many repeat signs it contains. This only applies to those equations in which all roots are real (Euler 1949, p. 468).

Now we refer to this rule as Descartes rule. Descartes himself claimed that, although he had Harriot's book (1631) at home, he only read it after he had finished his *Geometry (*1637). In Chapter 53, Wallis accused Descartes of borrowings from Harriot. In particular, Wallis highly appreciated the innovation of Harriot who suggested that all members of an equation be written as one member of the equality, setting them to zero.[26]

[21] This is the second publication. The first one was published in 1572.

[22] Leonard Digges (around 1515—around 1559), an English mathematician and topographer.

[23] "If I be not misinformed"—Wallis' note.

[24] Wallis meant *Francisci Vietae – in artem analyticem isagoge.*

[25] Wallis' book was in Euler's library in St. Petersburg.

[26] It should be noted that Harriot did not select this form of notation as the final one. In his records, the constant term is more often on the right side. One can come across an equation written with a zero in the right side as far back as in Stifel's works.

Wallis was right about the priority of Harriot. Descartes had Harriot's treatise when he was writing his *Geometry*. Many ideas which were thoroughly set forth and systematised by Descartes, had been first uttered by Harriot. It should be also noted that a book of A. Girard, *Invention nouvelle en l'Algèbre*, was published in Amsterdam in 1629 to formulate the basic theorem in algebra eight years before Descartes. However, Wallis did not mention this fact. Further, in Chapter 55 (Wallis 1685, p. 208), Wallis repeated that Descartes' reasoning was based on Harriot's *Algebra* published in 1631, while Descartes' *Geometry* was published in 1637 in French and thereafter, in 1649 and in 1659, in Latin. Wallis demonstrated that, although used by Viète and Bombelli, many procedures could be asserted much easier based on Harriot's *Algebra*. This, certainly, does not derogate Descartes' role, who, unlike the English, did not proceed from geometry to algebra. Instead, developing algebra and generalizing the notion of a number, he set the analytical direction in the development of geometry.

Wallis listed Harriot's achievements (up to 200-th page): symbols, terms, generation of equations by multiplying binomials, rule of signs (the number of positive and negative roots), methods of determining the number of real and imaginary roots, research of a quadratic equation, dividing an equation by a binomial, simplifying a cubic equation. Wallis acknowledged that almost all of these discoveries were made by Harriot, although some of them had been discovered earlier by Viète.

Wallis highly appreciated the role of Leonardo Pisano who reproduced Arab rules and symbols without resort to Diophantus who remained unknown in Europe until 1572.[27] According to Wallis, Stiefel was a good author who had never moved beyond quadratic equations (Wallis 1685, sheet a3). Scipione del Ferro, Cardano, Tartaglia, and other developed a solution of a cubic equation. Bombelli took it a step further, solving biquadratic equations (with the help of cubic ones,[28] reducing them to two quadratic equations). Nunes, Ramus, Schoener, Salignac,[29] Clavius, Record, T. Digges,[30] and some of our men (i.e. Englishmen—G.S.), were developing this subject in the last century. However, by and large, they had failed to take it a step further than the quadratic equations. At the same time, thanks to Xilander,[31] Diophantus became known in Latin, and thereafter, thanks to Bachet, in Greek and Latin[32]; all

[27] Bombelli found a manuscript of Diophantus in the library of Vatican and published 143 problems in his *Algebra*. Wallis was mistaken about symbols. Leonardus Pisanus had no symbols. Wallis had not seen Pisanus' works. He learnt about them in Pacioli's works. (Thanks to J. Cesiano for this remark).

[28] This is not true! It was Ferrari who created a formula to solve a biquadratic equation! However, Bombelli uttered nothing about Ferrari, although Ferreri's formula had been set forth in Cardanus' *Ars magna* – G.S.

[29] Johannes Salignacus, Scottish.

[30] Thomas Digges (1546–1595), son of Leonardo Digges, English mathematician and astronomer, one of the first partisans and promoters of the heliocentric world.

[31] Xilander published *Diophanti Alexandrini Rerum Arithmeticarum libri sex* in 1575 in Basel.

[32] Bachet de Méziriac published *Diophanti Alexandrini Arithmeticorum libri sex; et de Numeris multangulis liber unus. Nunc primum graece et latine editi, atque absolutissimis commentariis illustrati* in1621 in Paris.

his methods differed from Arab methods (followed by others). In particular, the procedure for naming exponents (Wallis 1685, sheet a3 verso): using new symbols and figures were an important step in algebra. The next major step in the development of algebra was made by François Viète in 1590 in his "specious arithmetic".[33] Wallis expressed his appreciation of this step. He noted that, unlike the preceding authors, in designating exponents, Viète adhered to Diophantus, not to Arabs.

Wallis' historical sketch was structured by subject matters. As a prominent mathematician, Wallis was by far more knowledgeable about mathematical information than other authors. He gave an unbiased account of the development of methods and discoveries in algebra and emerging analysis, although sometimes, one could, of course, blame him in subjective assessment. Unlike preceding authors, his narration was not a discrete set of biographies or discoveries. He showed mathematics as a continuous development of ideas and algebra in the first place. He demonstrated its internal relations and their continuity, genesis of mathematical knowledge, the creativity of mathematicians, but not their heuristic. He discerned algebraic and geometrical methods and distinguished the inception of an analytical method, i.e. *Differential Calculation* method, in works of his contemporaries and, first of all, of Newton. The Wallis book, perhaps, is the first historical and mathematical book, which outlines the history of mathematical thought.

12 1704, Edward Bernard

Edward Bernard (1638–1697) was a Savilian Professor[34] of astronomy in Oxford. He was connoisseur of ancient manuscripts; studied many manuscripts of Apollonius of Perga; worked in Bodleian Library (Oxford) with Arab manuscripts brought from Spain, Morocco, Syria, Arab countries, and Turkey, which, to a large extent, replenished its collection. Edward Bernard found an Arab text of Apollonius entitled *Determinate Section*; tried to recover those fragments that had been lost and translate them into Latin; he edited Josephus Flavius. Most of Edward Bernard's work consisted of annotating books from Bodleian Library: his work *On Ancient Weights and Measures (De mensuris et ponderibus antiquis*, 1688) was enclosed with a work of E. Pococke (1604–1691), an Orientalist scholar from Oxford. *Bernard's Catalogue* (Bernard 1697) comprised manuscripts from British and Irish libraries and served as a basic tool of scientists of that time. Many works of Bernard were not completed, which made his colleagues joke.[35] After Bernard 's death, his colleagues published a book about him (Huntington 1704, Sect. 9, pp. 1–78) which included Bernard's work

[33] So Wallis calls Viiet's "species logistic" this way.

[34] In 1619, Sir Henry Savile, mathematician, custodian of Merton College in Oxford and provost of Eton College, complaining of the "poor condition of mathematical research in England", constituted two positions to be funded at his own expense: professor of geometry and professor of astronomy, which are in existence to the present day. The first professor of geometry was Henry Briggs.

[35] E.g. epigram of Cl. Barksdale (1609–1687): "Savilian Bernard's a right learned man;/Josephus he will finish when he can".

entitled *A Short List of Ancient Greek, Latin, and Arab Mathematicians, prepared by Dr. Eduardo Bernardo, the most honoured and educated man* (Bernard 1704, Sect. 11, p. 1–44)—annotated plan of republishing oeuvres of classic authors kept in European archives and libraries, on circa 44 pages. The adaptations and translations of Apollonius' *Conics*[36] Bernard had made were subsequently used by Edmond Halley (1656–1742) in the 1710 publication of Apollonius' works.

13 1715, Joseph Raphson

In 1715, a small posthumous edition of Newton's disciple, Joseph Raphson,[37] was published. It was *History of Fluxions* (Raphson 1715), and the goal of this publication was to assert Newton's priority in the discovery of differential calculation. Newton allowed Raphson to look through his works and his correspondence with Leibnitz, and the respective presentment of this correspondence in Raphson's book provided a strong support to Newton's position in this dispute.

14 1741. Christian Von Wolff

Christian von Wolff (1679–1754), German philosopher, lawyer, and professor of mathematics, published a *Report on additions to mathematical sciences over one century* in Halley in 1707; *Mathematical Vocabulary* in Leipzig in 1716 (Wolff [1716] [1734] 1747)—a dictionary of mathematics in German, the best of those available by that time although not the first one, on 788 pages, the list of sources alone was on 54 pages; and an article entitled *A Summary of the Most Renowned Mathematical Works* in Volume V of *Elementary Fundamentals of Mathematical Sciences* (Wolfius 1741, pp. 3–168). In Chapter One (pp. 5–28), Wolff reviewed books, beginning with Euclid and finishing with publications of Academia Petropolitana until 1731. He, inter alia, mentioned works of young Euler. He devoted a paragraph with a summary to each of the above books. Those were books of French, English, and Dutch authors. Chapter Two (pp. 29–32) was devoted to the history of arithmetic from Nicomachus to Neper. Chapter Three (pp. 32–50), Geometry: Euclid and his translators, publishers, and annotators, European geometers, finishing with the year 1699. Chapter Four (pp. 51–69): analytical works from the ancient world to the inception of differential calculation. Much prominence was given to the dispute regarding

[36] *Conics, a* fundamental treatise of Apollonius of Perga consisted of eight books. The Greek text of four of them has been preserved; three other books have survived translated into Arabic; the eighth book was renovated in the eighteenth century by E. Halley who published Apollonius' works (Oxford, 1710).

[37] Joseph Raphson, an English mathematician, Newton's disciple, died before 1715. We know very little about his life. He was the author of the most appropriate statement of Newton's approximation approach.

the priority of Newton and Leibnitz. And Wolff, professional lawyer and Leibnitz' friend, gave preference to Leibnitz in this matter, arranging their correspondence and publications chronologically, without getting involved in the mathematical substance. Chapter Five (pp. 71–77): trigonometry from Ptolemy to Ozanam. Chapters Six to Thirteen were devoted to statics, mechanics (up to Euler), hydrostatics, aerometry, hydraulics, optics, catoptric, dioptric, perspective projection, astronomy, chronology, geography, gnomonics, civil architecture, pyrotechnics, and military architecture—well-established sections of the eighteenth century mathematics. The book contains an index of names.

15 1742, Johann H. Heilbronner

The last book in the early period of historiography (before Montucla) was published in 1742. This was *The History of Mathematics at Large—from the Creation to the 16th Century A.D. including life stories of famous mathematicians, their doctrines, works, and manuscripts; in addition, a summary of main mathematical collections and works, and history of arithmetic to the present day* (Heilbronner 1742) in Latin. Its author was a German theologian and mathematician Johann Heilbronner (1706–1745/1747). The book contained an index of names. It was a bulky book—924 pages. Montucla called this Heilbronner's work "chaos" (Popov 1920, pp. 152–153). The author paid much attention to philosophical issues and described the structure of mathematics. He presented the sequence of certain names and discoveries in mathematics in considerable detail, although not free from errors. He listed famous manuscripts and published books. This *Historia matheseos* compared favourably with the previous books thanks to the two special features. First, the author added modest information from the history of Arab and Chinese mathematics (names and discoveries) to the European history; second, all these different national histories were reduced to a single time scale. Heilbronner used the achievements in chronology of the last century and dated each event in mathematics in several ways: mentioned the eclipses which happened at that time or other celestial events, specifying their characteristics from astronomical tables (of Ptolemy and other astronomers), year anno mundi (ad annum Mundi), year from the founding of the City of Rome (ab urbe condita), year B.C. (ante Christum natum, ante Christi nativitatem), or year A.D. (ab Anno Christi). This presentation was not infallible. E.g. on page 353, Thang-Heng,[38] Chinese mathematician and astronomer, was dated to the year 164. Heilbronner carried Michael Psellos (eleventh century) back to the ninth century (p. 410), while Al-Farabi (872–950) and Ibn Musa (al-Khwarizmi, around 820) were dated to the tenth century. However, regardless of numerous disadvantages, it was in Heilbronner's book that the image of the history of world mathematics appeared for the

[38] Heilbronner used to be very reliant on the letters about the history of Chinese astronomy of Antoine Gaubil, cartographer and missionary in China, which were published in European journals as of 1729.

first time ever, combining histories of different cultures. After Heilbronner's death, his library was purchased by Kästner who wrote a history of mathematics of his own. However, this is a topic for another article.

16 Coda

The list of the books provided can be supplemented. A good review can be found in Popov's book (Popov 1920), although one could blame him of some inaccuracies and gaps. However, the author was very scrupulous writing his book, he had read all the books he wrote about. A book devoted to historical development of historiography and mathematics in various countries was published in 2002 (Dauben and Scriba 2002). However, the period till 1750 was illustrated in it all too briefly. Thus, in the first two millennia of its existence, the history of mathematics began developing scientific methodology: performing scientific analysis of works, sources (original, translated, renderings, and annotations); distinguishing between facts and interpretations; drafting catalogues and reference books; issues relating to individual and collective authorship (of national school), analysing the application and teaching of mathematical methods; chronology. In addition, textual analysis was emerging, the purpose of writing of mathematical works was not identified (research, teaching). Authors did not consider the role and reciprocal influence of ancient civilizations—as a rule, they began the history of mathematics from Greeks and considered it mostly in Latin culture. Arab manuscripts began to become the custom; Chinese manuscripts were hardly mentioned; and Indian manuscripts were almost unknown. The issues of national priorities were solved quite simply—each historian knew mathematical literature of his own country pretty well, giving preference to his compatriots (as, for example, Wallis or Wolff). This was the way the historical and mathematical memory of the nation consolidated and its mentality shaped. Absolute chronology was evolving up to the seventeenth century. Therefore, mathematical achievements in the social time of different civilisations only began to be compared. They did not distinguish between the development stages, periods of boom and bust, they did not give prominence to the trends in the evolution of mathematics, they did not emphasise its independence or the degree of its dependence on the needs of that time. Gradually, the presentation of the history of mathematics was undergoing stages – from describing discoveries and biographies by type of chronicles to genesis of ideas and understanding of mathematical progress.

17 Conclusions

As a science, the history of mathematics was evolving together with the evolution of mathematics itself. Ancient works described the sequence of discoveries in mathematics based on the principle 'who discovered what', 'who taught whom', 'who followed whom'. Eudemus of Rhodes compared works that were topically related. Proclus began distinguishing research from educational works. With Dionysius de Laerte, biographies appeared in the history of mathematics. However, the history of mathematics was confined in a single culture.

The phenomenon of scientific translation as an art and its systematisation appeared in Arab culture of the 8th–twelfth centuries. Thanks to Arabic translations, the ancient heritage was preserved. This tradition was carried on in the Christian medieval period: Greek, Byzantine, and Arabic texts were translated into Latin; compendiums came into being—those were abridged renderings of classic authors which contained historical information. In Muslim culture of the 14th–seventeenth centuries, great importance was attached to accounting and classifying manuscripts, describing thereof providing information from their authors' biographies; first catalogues began to be created. As of the twelfth century, Englishmen began collecting manuscripts brought to England from the East; research libraries emerged, e.g. the Bodleian library (fourteenth century). The advent of book printing (fifteenth century) gave an impulse to dissemination of works of ancient classic authors; they were annotated and discussed, which included criticism as well. They began accounting each issue of a book as an individual scientific event. However, until the sixteenth century, the entire body of mathematical knowledge appeared to be static, lacking development (Ramus), although it was consistently establishing in time. Only the development of teaching methods was considered. It was the seventeenth century that a notion of progress in mathematics appeared for the first time in Voss' works. The history of creation gradually replaced the history of discoveries.

Chronologically, research works were arranged in an ordered fashion inside each culture: the chronology by Olympiad, since the creation of Rome; Muslim chronology—from Hegira; Chinese, by dynasty; Christian, Anno Domini, etc. Events, including mathematical events, which happened in various cultures were not related to one another; the story of each culture was stated independently. Wallis, for example, believed that Arabs knew algebra: "After Diophantus, not to say before him, the notion of exponentiation was studied by Arab authors. However, they remained unaware of it in Europe for a rather long time." (Wallis 1685, pp. 4–5). This also resulted in "national shortsightedness", when discoveries of compatriots seemed closer and more important (e.g., Wallis and Wolff).

Thanks to the works of Scaliger and Petavius, the chronology reform of the 16th–seventeenth centuries made it possible to bring historical events in compliance with astronomical phenomena and reduce them to a single scale which had a starting point and a direct reading-scale and a countdown-scale (A.D., B.C.). It should be noted that by this time, zero began to be apprehended as a reference point and a negative number, as a possibility of a countdown of steps, time, temperature (Wallis in the seventeenth century, Celsius in the eighteenth century). Authors began to include not only Christian mathematicians in the history of mathematics, Muslim mathematicians were mentioned as well (e.g., Baldi and Blankanus).

Methods used in other historical sciences began to be used in the history of mathematics. Those were methods of paleography, historical criticism, chronology, the doctrine of historical sources, evidence of their authenticity, ways to identify forgery, ancient written instruments and materials, styles. In the eighteenth century, name indices began to appear in books.

Authors began to try and present the history of mathematics not as a chronicle but as a history of ideas (Buteo in the sixteenth century, then Wallis in the seventeenth century).

All this made the history of mathematics ready for its next fledging period which began in 1758 when Montucla's History of Mathematics appeared.

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